

Physics 681: Solar Physics and Instrumentation – Lecture 21

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Magnetism

- Fields and Conducting Matter
 - The Induction Equation
 - Electrical Conductivity on the Sun
 - Frozen Magnetic Field
 - The Magnetic Force
- Flux Tubes
 - Concentration of Magnetic Flux Tubes
 - Observational Evidence for Flux Tubes
 - Vertical Thin Flux Tubes
 - Curved Thin Flux Tubes
 - Thermal Structure of Photospheric Tubes
- Sunspots
- The Solar Cycle

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Fields and Conducting Matter

- Magnetic features: sunspots, sunspot fine structure, filaments, prominences, ...
- Magnetism on the Sun occurs on all scales! (small flux tubes to mean field)
- Maxwell's equations (magnetic permeability $\mu = 4\pi \times 10^{-7}$ Vs/Am)

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu \mathbf{j}$$

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

- Ohm's law

$$\tilde{\mathbf{j}} = \sigma \tilde{\mathbf{E}} \quad \text{with } \tilde{\mathbf{j}} = \mathbf{j} \quad \text{and } \tilde{\mathbf{E}} = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

$$\Rightarrow \mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



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- Induction equation

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

- Magnetic diffusivity

$$\eta = \frac{1}{\mu\sigma}$$

- Magnetic Reynolds number

$$R_m = \frac{vl}{\eta} = \frac{\tau_D}{\tau_A} \quad \text{with } \tau_D = \frac{l^2}{\eta} \quad \text{and } \tau_A = \frac{l}{v}$$

- High conductivity: $R_m \gg 1$ or $\tau_D \gg \tau_A$

- Electrical conductivity of a fully ionized gas

$$\sigma = \frac{32\varepsilon_0^2 \sqrt{\pi} (2kT)^{3/2}}{\sqrt{m_e} e^2 Z \ln \Lambda} \gamma_E \quad \text{with } \Lambda = \frac{r_D}{p_0}$$



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- In $\Lambda \approx 5$ for the solar interior, In $\Lambda \approx 10$ for the chromosphere, and In $\Lambda \approx 20$ for the corona
- Conductivity is large:

$$\sigma \approx 0.003 T^{3/2} \text{ A/Vm}$$
- The temperature in the solar photosphere/chromosphere is low \Rightarrow hydrogen and helium are neutral BUT heavier elements with low ionization potentials still donate electrons \Rightarrow collisions of electrons with neutrals \Rightarrow conductivity is greatly reduced
- Conductivity for weak ionization

$$\sigma = \frac{3e^2}{8S(2\pi m_e kT)^{1/2}} \frac{n_e}{n_n}$$

- Large horizontal gradients (e.g., sunspots)
- Collisions are rare and magnetic fields are strong \Rightarrow anisotropy
- Plasma turbulence reduces conductivity



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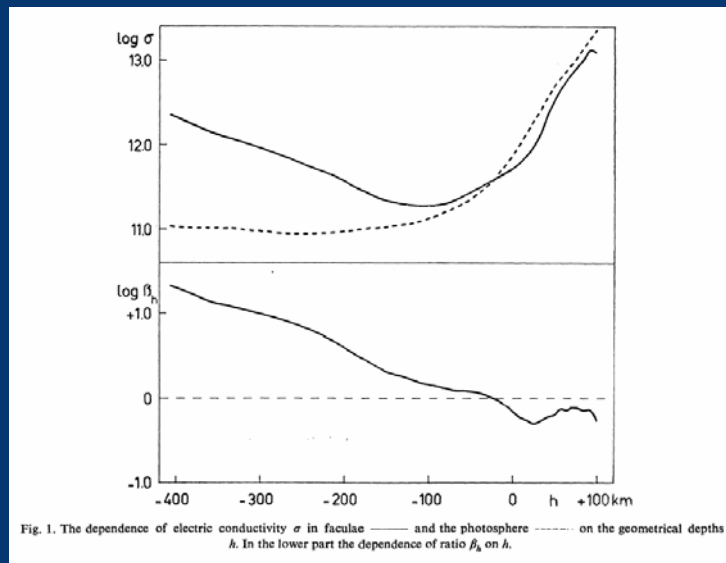


Fig. 1. The dependence of electric conductivity σ in faculae — and the photosphere - - - on the geometrical depths h . In the lower part the dependence of ratio β_h on h .

Kopecký and Soytürk (1971)



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Frozen Magnetic Fields

- High conductivity $\sigma = 1 \text{ V/A}$ (in sunspots), typical velocity $v = 1 \text{ km/s}$, and scale height $l = 100 \text{ km} \Rightarrow$ frozen field lines
- Total magnetic flux

$$\Phi = \int_F \mathbf{B} \cdot d\mathbf{f}$$

- Differential flux at two instances separated by an infinitesimal time interval dt

$$\Phi' - \Phi = dt \left[\int_F \frac{d\mathbf{B}}{dt} \cdot d\mathbf{f} + \int_S \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{s}) \right]$$

$$\text{with } \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{s}) = (\mathbf{B} \times \mathbf{v}) \cdot d\mathbf{s} \quad \text{and} \quad \int_S (\mathbf{B} \times \mathbf{v}) \cdot d\mathbf{s} = - \int_F [\nabla \times (\mathbf{v} \times \mathbf{B})] \cdot d\mathbf{f}$$

$$\Rightarrow \frac{d\Phi}{dt} = \int_F \left[\frac{d\mathbf{B}}{dt} - \nabla \times (\mathbf{v} \times \mathbf{B}) \right] \cdot d\mathbf{f} \stackrel{\sigma \rightarrow \infty}{=} 0$$



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