

# Physics 681: Solar Physics and Instrumentation – Lecture 19

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## Oblateness

- A rotating, non-rigid body must have an oblateness
- Consider an inviscid star with rigid rotation  
 $\mathbf{v} = (0, 0, r\Omega \sin \theta)$
- Equilibrium in a frame of rest  
 $\rho \mathbf{v} \nabla \mathbf{v} = -\nabla P - \rho \nabla \Phi$
- Constant pressure  $P$  at the surface  
 $\Phi - \frac{1}{2}(r\Omega \sin \theta)^2 = \text{const.}$
- Continuous transition of  $\Phi$  to the outer gravitational potential

$$\Phi = \Phi_{\text{ext}} \equiv -\frac{Gm_{\odot}}{r} \left[ 1 - J_2 \left( \frac{r_{\odot}}{r} \right)^2 P_2(\theta) \right]$$



- Approximation of the oblate surface ignoring differential rotation while including a quadrupole moment arising from a more rapidly rotating core as compared to the surface rotation

$$r(\theta) = r_{\odot} (1 - cP_2(\theta)) \quad \text{and}$$

$$\Delta r / r_{\odot} \equiv (r_{\text{equator}} - r_{\text{pole}}) / r_{\odot} = 3c/2 \quad \text{with} \quad \sin^2 \theta = 2(1 - P_2)/3$$

$$\Rightarrow \frac{1}{3} (\Omega r_{\odot})^2 (1 - P_2) + \frac{Gm_{\odot}}{r_{\odot}} (1 + cP_2) - \frac{Gm_{\odot} J_2 P_2}{r_{\odot}} = \text{const.}$$

$$\Rightarrow \frac{\Delta r}{r_{\odot}} = \frac{1}{2} \frac{\Omega^2 r_{\odot}}{g_{\odot}} + \frac{3}{2} J_2 \quad \text{with} \quad g_{\odot} = \frac{Gm_{\odot}}{r_{\odot}^2}$$

- Synodic angular velocity (Carrington):  $\Omega_{\text{syn}} = 2.67 \times 10^{-6} \text{ Hz}$
- Sidereal angular velocity:  $\Omega_{\text{sid}} = 2.87 \times 10^{-6} \text{ Hz}$
- Oblateness:  $1.04 \times 10^{-5}$  or 14 km or 0.02 arcsec
- The oblateness is difficult to measure!
- Perihelion of Mercury & General Theory of Relativity
- Oblateness seems to be related to only the surface rotation



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## Oblateness of Jupiter

- Equatorial radius:  $R_e = 71,370 \text{ km}$
- Polar radius:  $R_p = 66,750 \text{ km}$
- Oblateness:  $(R_e - R_p) / R_e = 0.0648$
- First order correction term in gravitational potential:  $\Phi \equiv U/m$

$$\Phi(\theta) = -\frac{GM}{r} \left[ 1 - \left(\frac{R_e}{r}\right)^2 J_2 P_2(\cos \theta) - \left(\frac{R_e}{r}\right)^4 J_4 P_4(\cos \theta) - \dots \right]$$

$$P_0(\cos \theta) = 1$$

$$P_2(\cos \theta) = \frac{1}{2} (3 \cos^2 \theta - 1)$$

$$P_4(\cos \theta) = \frac{1}{8} (34 \cos^4 \theta - 30 \cos^2 \theta + 3)$$

Legendre Polynomials

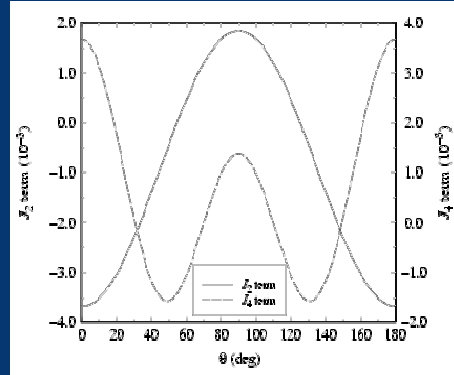
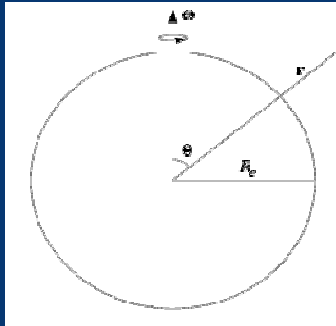


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# Gravitational Moments

- $J_2$ : oblateness and moment of inertia
- $J_4$ : mass distribution in outer regions, equatorial bulge, and planets thermal structure



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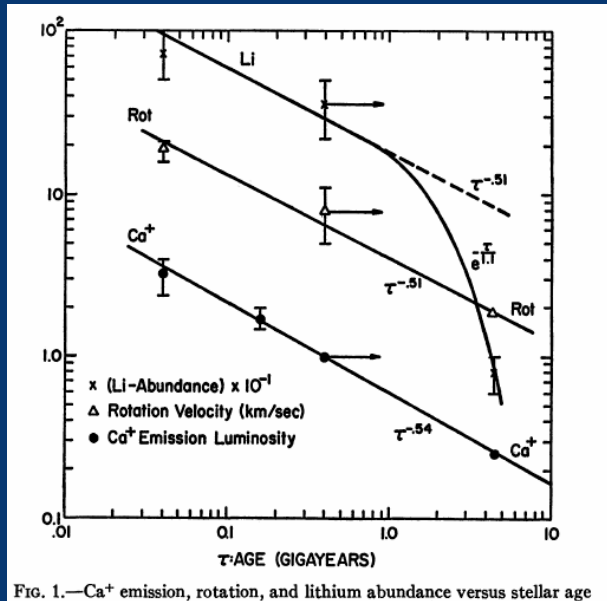
# Rotational History

- Determine the evolution of a rotating star from its initial angular velocity (T Tauri stars  $\approx 15$  km/s)
- The initial angular momentum of the Sun was much larger than that of the whole present solar system
- Magnetic breaking!
- The rotation rate of main-sequence stars similar to the Sun decreases with age (stellar activity  $\Rightarrow$  Ca II H & K emission)
- Stars earlier than F5 have no deep outer convection zone  $\Rightarrow$  no magnetic field generation  $\Rightarrow$  no breaking  $\Rightarrow$  rapid rotators (O to F stars rotate up to 100 times faster than the Sun)
- Pre-main-sequence solar models are fully convective
- Turbulent friction leads to a uniform angular velocity
- Total angular momentum:  $J_0 = 5 \times 10^{42}$  to  $5 \times 10^{43}$  km<sup>2</sup>/s (more than 260 times today's value)

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Skumanich (1972)



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## Torques

- Magnetic braking
- Material escaping the rotating solar surface carries some angular momentum with it
- The magnetic lines of force act as a lever arm that forces the escaping material to rotate rigidly with the solar surface far out to the Alfvén radius  $r_A$
- Beyond  $r_A$  the magnetic field is too weak to enforce rigid rotation
- Total loss of angular momentum

$$\frac{dJ}{dt} = \Omega r_A^2 \frac{dm}{dt}$$

- Parameterization of torque by a power law

$$\frac{dJ}{dt} = K \Omega^\alpha$$



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- ❑ Angular momentum transport in the interior
- ❑ Uniform rotation is maintained as long as the Sun was fully convective
- ❑ Internal torque required for angular momentum transport is provided by turbulent friction  $\Rightarrow$  slowing the Sun as a whole in the early phase of the evolution
- ❑ Development of a radiative core  $\Rightarrow$  only the outer convective shell rotates uniformly and loses angular momentum
- ❑ Core contracts and rotates more rapidly
- ❑ Splitting of  $p$ -mode frequencies? Oblateness? Stability?
- ❑ No evidence for a fast rotating core!
- ❑ Instabilities in the presence of strong shear motion!
- ❑  $\Rightarrow$  Internal magnetic torque
- $B_p \approx (\mu\rho)^{1/2} r/t \geq 10^{-10} \text{ T}$
- ❑ Magnetic torque also acts as a restoring force of torsional oscillations



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## Evolution of Solar Rotation

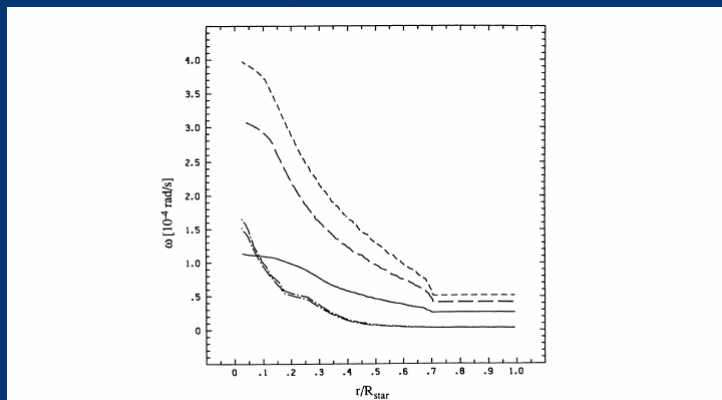


FIG. 4.—Angular velocity as a function of radius at two ages for three models differing only in their initial angular momentum  $J_0$ . The solid line is the reference solar model (case A in Table 2) with  $J_0 = 5 \times 10^{48} \text{ g cm}^2 \text{ s}^{-1}$  at an age of  $3 \times 10^7 \text{ yr}$ . The long-dashed line is a model with  $J_0 = 1.63 \times 10^{49} \text{ g cm}^2 \text{ s}^{-1}$  (case B) at the same age and the short-dashed line is a model with  $J_0 = 5 \times 10^{48} \text{ g cm}^2 \text{ s}^{-1}$  (case C) at the same age. The dash-dotted lines indicate the range of rotation curves in the present-day Sun. An order of magnitude variation in  $J_0$  produces a large change in the rotation curve at an early age but only a small change in the rotation curve of the present-day Sun.

Pinsonneault et al. (1989)



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