Physics 681: Solar Physics and Instrumentation – Lecture 15

Carsten Denker

NJIT Physics Department

Center for Solar–Terrestrial Research

Convection

- Stability
- Mixing-Length Theory
  - The Local Formalism
  - Numerical Test Calculations
  - Overshooting: A Non-Local Formalism
- Granulation
- Mesogranulation
- Supergranulation
- Giant Cells

Stability

- Energy transport mechanisms: radiation, convection, and conduction
- Stratification of the outer layer (≈200,000 km) is unstable
- Mixing-length concept: parcel of gas travels a certain distance, dissolves, and deposits its energy
- Adiabatic displacement from equilibrium position (no oscillation!)
- Schwarschild criterion (1906)
- Condition for instability (pressure equilibrium)

$$\rho^* - \rho_0^* = \delta r \left[\left(\frac{d\rho}{dr} \right)_a - \frac{d\rho}{dr} \right] < 0$$

- Variations of the mean molecular weight μ and chemical composition

$$\frac{dT}{dr} < \left(\frac{dT}{dr} \right)_a + \frac{T}{\mu} \frac{d\mu}{dr} - \frac{T}{\mu} \left(\frac{d\mu}{dr} \right)_a$$

- Ideal gas equation

$$P = \rho \mathcal{R} T / \mu$$

- Core material is fully ionized $(d\mu/dr)_a = 0$
- $d\mu/dr$ towards center is negative due to the accumulation of helium ⇨ stabilizing
- Assume instantaneous adjustment of the ionization equilibrium

$$\frac{d\mu}{dr} = \left(\frac{\partial \mu}{\partial P} \right)_T \frac{dP}{dr} + \left(\frac{\partial \mu}{\partial T} \right)_P \frac{dT}{dr} \quad \text{and}$$

$$\frac{d\mu}{dr} = \left(\frac{\partial \mu}{\partial P} \right)_T \left(\frac{dP}{dr} \right)_a + \left(\frac{\partial \mu}{\partial T} \right)_P \left(\frac{dT}{dr} \right)_a$$

- Schwarzschild criterion

$$\frac{dT}{dr} < \left(\frac{dT}{dr} \right)_a$$

$$\nabla > \nabla_a \quad \Leftrightarrow \quad \frac{d \ln T}{d \ln P} > \left(\frac{d \ln T}{d \ln P} \right)_S$$

Mixing-Length Theory

- Viscosity vs. turbulent motion
- Numerical methods (Navier-Stokes equation in 3D)
- Schmidt and Prandtl (1915-1930) and Böhm-Vitense (1958)
- Mixing-length

  $l = \alpha H_{\mathrm{P}}$
- Energy flux

  $F_{\mathrm{R}} + F_{\mathrm{C}} = L_{\odot} / 4\pi r^2$
- Radiative flux

  $$F_{\mathrm{R}} = -\frac{16\sigma T^3}{3\kappa\rho}\frac{dT}{dr} = \frac{16\sigma T^4}{3\kappa\rho H_{\mathrm{P}}}\nabla$$
- Mean excess temperature

  $$\Delta T = \left[\left(\frac{dT}{dr}\right)' - \frac{dT}{dr}\right]\delta r \equiv \left(\nabla - \nabla'\right)\frac{T\delta r}{H_{\mathrm{P}}}$$

- Assume that the mean value of δr is half the mixing length

  $\Delta T = \left(\nabla - \nabla'\right) T\alpha / 2$
- Convective flux

  $F_C = \alpha\rho c_{\mathrm{P}} v T\left(\nabla - \nabla'\right)/2$
- Convective velocity

  $\partial^2\delta r / \partial t^2 = -g\Delta\rho / \rho = g\delta\Delta T / T$
- Pressure equilibrium ($\Delta P = 0$) and integration

  $$\left(\frac{\partial\delta r}{\partial t}\right)^2 = \frac{g\delta}{H_{\mathrm{P}}}\left(\nabla - \nabla'\right)\left(\delta r\right)^2$$
- Work done by buoyancy force appears as kinetic energy of the parcel
- Introduce correction factor of ½ to account for friction

  $$v = l\left[\frac{g\delta}{8H_{\mathrm{P}}}\left(\nabla - \nabla'\right)\right]^{1/2}$$

- Radiative losses across the surface of the parcel (d = distance over which ΔT drops to zero)

$$F'_{\mathrm{R}} = -\frac{16\sigma T^3}{3\kappa\rho}\frac{\Delta T}{d} = \frac{8\alpha\sigma T^4}{3\kappa\rho d}\left(\nabla' - \nabla\right)$$

- Convective flux

$$F_{\mathrm{C}} = \alpha\rho c_{\mathrm{P}} vT\left(\nabla - \nabla_a\right)/2 + \alpha\rho c_{\mathrm{P}} vT\left(\nabla_a - \nabla'\right)/2$$

- First term (ideal adiabatic conditions) and second term (real convective flux)
- Radiative loss per unit time

$$\frac{16\sigma T^3 S}{3\kappa\rho d}\left(\nabla' - \nabla\right) = \rho c_{\mathrm{P}} vq\left(\nabla_a - \nabla'\right)$$

- Spherical parcels d = (8/9) l and Sl / qd = 9/2
- Temperature gradients

$$\nabla' - \nabla_a = 2U\left(\nabla - \nabla'\right) \quad \text{with} \quad U = \frac{24\sqrt{2}\sigma T^3 P^{1/2}}{c_{\mathrm{P}}\kappa g l \delta^{1/2}\rho^{5/2}}$$

Conditions for Convection in Stars

- The opacity is large, implying that an unachievable temperature gradient would be necessary for radiative energy transport
- A region exists where ionization is occurring, causing a large specific heat and a low adiabatic temperature gradient
- The local gravitational acceleration is, as would be the case in distended stars, again leading to a low adiabatic temperature gradient
- The temperature dependence of the nuclear energy generation rate is large, causing a steep radiative flux gradient and a large temperature gradient (CNO cycle or triple alpha process)