

# Physics 681: Solar Physics and Instrumentation – Lecture 11

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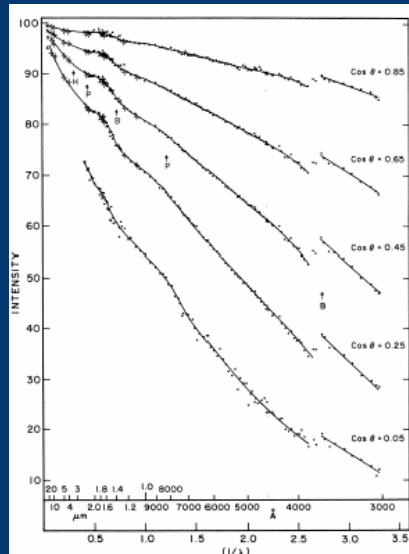
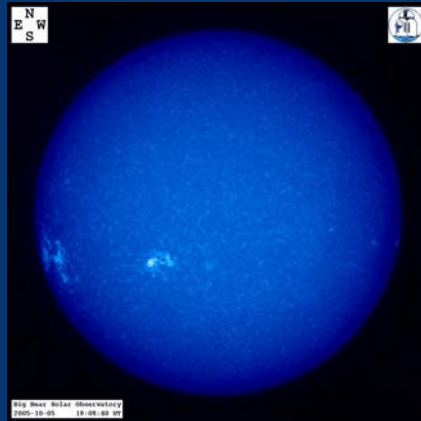


Fig. 2. Limb darkening as a function of reciprocal wavelength for selected  $\cos \theta$  values. The short arrows labeled  $\alpha, \beta, \dots$ , represent the Balmer, Paschen, etc., limits.

Pierce, Slaughter, & Weinberger (1977)

## Limb Darkening



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# Non-LTE Spectral Line Formation

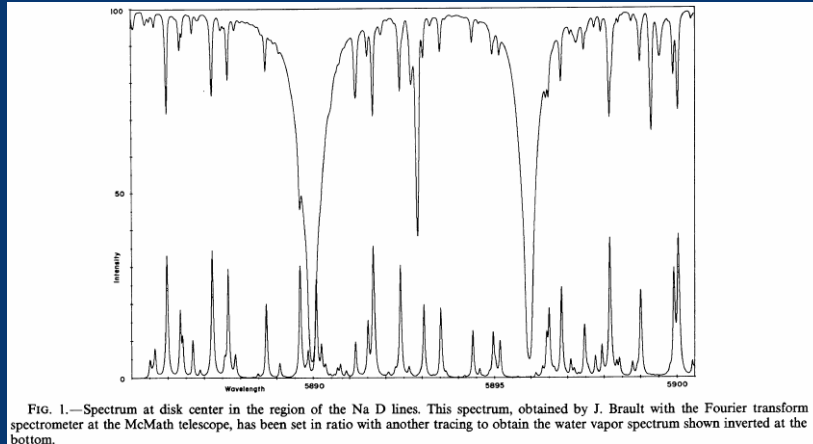


FIG. 1.—Spectrum at disk center in the region of the Na D lines. This spectrum, obtained by J. Brault with the Fourier transform spectrometer at the McMath telescope, has been set in ratio with another tracing to obtain the water vapor spectrum shown inverted at the bottom.

Pierce & Slaughter (1982)



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# Source Function

- Radiative transfer equation

$$\mu \frac{dI_\nu}{dr} = -\kappa \rho I_\nu + \varepsilon \quad \text{with } \kappa = \kappa_l + \kappa_c \quad \text{and } \varepsilon = \varepsilon_l + \varepsilon_c$$

- Net gain of the intensity for line radiation

$$\frac{h\nu}{4\pi} \left[ n_U (A_{UL}\chi + B_{UL}I_\nu\psi) - n_L B_{LU}I_\nu\phi \right]$$

- Line absorption coefficient and line source function

$$-\rho\kappa_l I_\nu + \varepsilon_l \quad \text{and} \quad S_l = \varepsilon_l / \rho\kappa_l \quad \Rightarrow$$

$$\kappa_l = \frac{h\nu}{4\pi\rho} (n_L B_{LU}\phi - n_U B_{UL}\psi) \quad \text{and}$$

$$S_l = \frac{n_U A_{UL}\chi}{n_L B_{LU}\phi - n_U B_{UL}\psi}$$



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- Complete redistribution (exact coherence/complete incoherence)

$$\chi(\nu) = \psi(\nu) = \phi(\nu)$$

- Frequency independent line source function

$$S_l = \frac{n_U A_{UL}}{n_L B_{LU} - n_U B_{UL}}$$

- Continuum contribution

$$-\rho \kappa_{Cj} I_\nu + \varepsilon_{Cj} \equiv \alpha_j(\nu) \left[ n_C e^{-h\nu/kT} \frac{n_j^*}{n_C^*} \left( \frac{2h\nu^3}{c^2} + I \right) - n_j I_\nu \right]$$

- Continuum absorption coefficient and continuum source function

$$\kappa_{Cj} = \rho^{-1} \alpha_j(\nu) \left( n_j - n_C e^{-h\nu/kT} \frac{n_j^*}{n_C^*} \right)$$

$$S_{Cj} = \frac{2h\nu^3}{c^2} \frac{1}{\left( n_j / n_C \right) \left( n_j^* / n_C^* \right) \exp(h\nu/kT) - 1}$$



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- LTE ratios of the levels  $j$  and  $C$

$$\frac{n_j^*}{n_C^*} = \left( \frac{h^2}{2\pi m_e kT} \right)^{3/2} \frac{n_e g_j}{2u_C} \exp\left( \frac{E_C - E_j}{kT} \right)$$

- Combine continuum and line radiations (for absorption sum all  $\kappa$ 's)

$$S = \frac{\kappa_l S_l + \kappa_C S_C}{\kappa_l + \kappa_C}$$

- Statistical Equilibrium

$$n_L (B_{LU} \bar{J}_{LU} + C_{LU}) = n_U (A_{UL} + B_{UL} \bar{J}_{UL} + C_{UL}) \quad \text{with}$$

$$\bar{J}_{LU} = \bar{J}_{UL} = \frac{1}{4\pi} \int I_\nu \phi(\nu) d\nu d\Omega$$

- Abundance relative to hydrogen

$$n_L + n_U = \varepsilon n_H$$



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- Population of the  $j$ th level

$$n_j \sum_{i \neq j} R_{ji} = \sum_{i \neq j} n_i R_{ij} \quad \text{with} \quad R_{ji} = A_{ji} + B_{ji} \bar{J}_{ji} + C_{ji}$$

- Transitions from level  $j$  to the continuum

$$R_{jC} = 4\pi \int_{\nu_{jC}}^{\infty} \frac{\alpha_j(\nu)}{h\nu} J_{\nu} + C_{jC} \quad \text{with} \quad J_{\nu} = \frac{1}{4\pi} \int I_{\nu} d\Omega$$

- Recombination rate

$$R_{Cj} = 4\pi \frac{n_j^*}{n_C^*} \int_{\nu_{jC}}^{\infty} \frac{\alpha_j(\nu)}{h\nu} e^{-h\nu/kT} \left( \frac{2h\nu^3}{c^2} + J_{\nu} \right) d\nu + C_{Cj}$$

- Abundances relative to hydrogen

$$\sum_j n_j = \epsilon n_H$$

- Departure coefficients  $b_j = \frac{n_j / n_j^*}{n_C / n_C^*}$  and  $b_C = 1$



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## Atmospheric Models

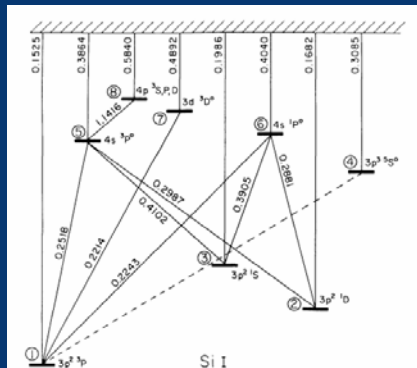


Fig. 2.—The energy-level diagram for our model of the Si I atom.

Vernazza, Avrett, & Loeser (1976)

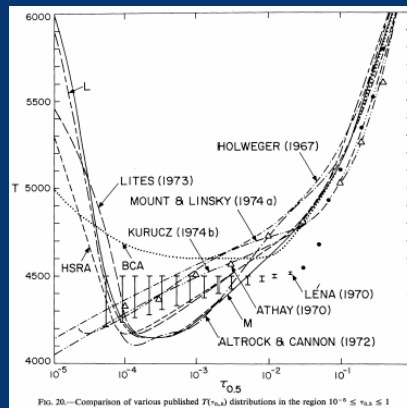


Fig. 20.—Comparison of various published  $T(\nu_{jC})$  distributions in the region  $10^{-5} \leq \nu_{jC} \leq 1$ .



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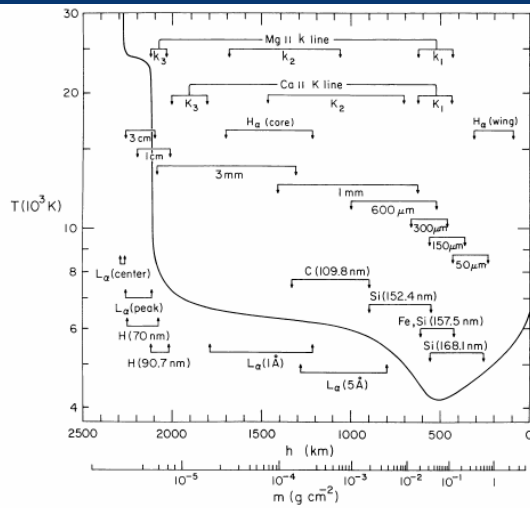


FIG. 1.—The average quiet-Sun temperature distribution derived from the EUV continuum, the  $L\alpha$  line, and other observations. The approximate depths where the various continua and lines originate are indicated.

Vernazza, Avrett, & Loeser (1981)



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