Physics 681: Solar Physics and Instrumentation – Lecture 9

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Polarimetry

- Zeeman splitting
- Assume weak magnetic field (LS or Russel-Saunders coupling)
- Landé factor

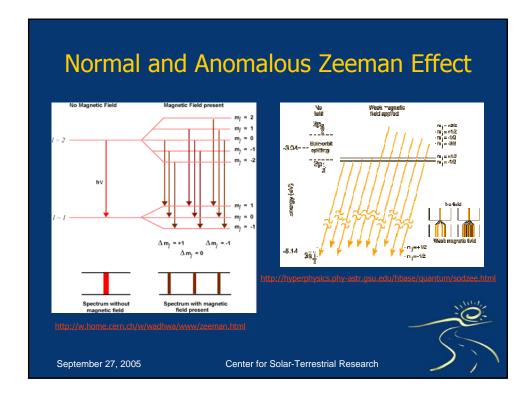
$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

□ Displacement of the line in the presence of a magnetic field

$$\Delta \lambda_B = \lambda - \lambda_0 = \frac{e}{4\pi c m_e} g^* \lambda^2 B$$
 with $g^* = gM - g'M'$

□ Normal Zeeman effect or Lorentz triplet: S = S = 0 $\Delta M_7 = -1, 0, +1 \Rightarrow g^* = -1, 0, +1$

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- Solar spectral lines are broadened by micro- and macro-turbulence and by pressure

 ⇒ the components of a Zeeman multiplet are normally not resolved
- $lue{}$ The triplet consists of two shifted σ -components and an unshifted π -
- □ Longitudinal Zeeman effect: line-of-sight (LOS) || *B* \Rightarrow σ -components with circular polarization of opposite sense
- □ Transverse Zeeman effect: LOS \perp $B \Rightarrow r$ -component (linearly polarized $\perp B$) and σ -components (linearly polarized $\mid B$)
- \square $\Delta \lambda_B \approx \lambda^2 \Leftrightarrow \Delta \lambda_D \approx \lambda$
- □ Polarized light (propagating in the *z*-direction)

$$E_x = \xi_x \cos \phi$$
 and $E_y = \xi_y \cos (\phi + \varepsilon)$

$$I = \xi_{\rm r}^2 + \xi_{\rm v}^2$$
 $Q = \xi_{\rm r}^2 - \xi$

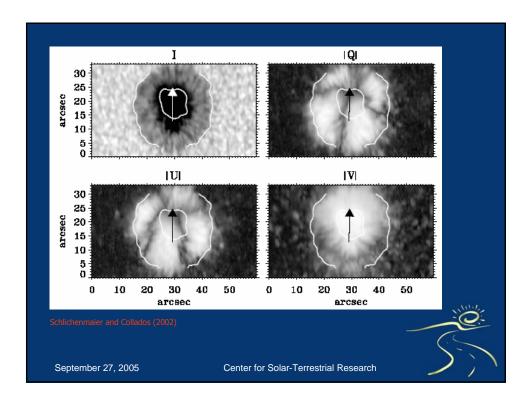
Stokes vector
$$I^{2} = Q^{2} + U^{2} + V^{2}$$

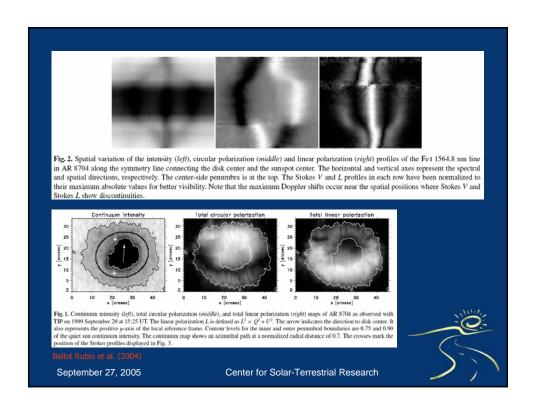
$$I = \xi_{x}^{2} + \xi_{y}^{2} \qquad Q = \xi_{x}^{2} - \xi_{y}^{2}$$

$$U = 2\xi_{x}\xi_{y}\cos\varepsilon \qquad V = 2\xi_{x}\xi_{y}\sin\varepsilon$$

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□ Superposition of a large number of independent waves

$$I = \left\langle \xi_x^2 + \xi_y^2 \right\rangle \qquad Q = \left\langle \xi_x^2 - \xi_y^2 \right\rangle$$

$$U = 2 \left\langle \xi_x \xi_y \cos \varepsilon \right\rangle \quad V = 2 \left\langle \xi_x \xi_y \sin \varepsilon \right\rangle$$

Degree of polarization

$$P = \left(\frac{Q^2 + U^2 + V^2}{I^2}\right)$$

- □ Stokes profiles $I(\lambda)$, $Q(\lambda)$, $U(\lambda)$, and $V(\lambda)$
- Longitudinal Zeeman triplet

$$\begin{split} I &= I_{c} \left(1 - \tau \right) - I_{c} \tau \left(\eta^{+} + \eta^{-} \right) / 2 \\ \eta^{\pm} &= \eta \left(\lambda \pm \Delta \lambda_{B} \right) \text{ with } \eta \left(\lambda \right) = \kappa_{l} \left(\lambda \right) / \kappa_{C} \\ V \left(\lambda \right) &= -I_{c} \tau \left(\eta^{+} - \eta^{-} \right) / 2 \end{split}$$

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Unno's Equations

 $\Delta \boldsymbol{I} = -\tau \boldsymbol{I} - \tau \eta \boldsymbol{I}$ with

$$\boldsymbol{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \quad \boldsymbol{\eta} = \begin{pmatrix} \eta_{I} & \eta_{Q} & \eta_{U} & \eta_{V} \\ \eta_{Q} & \eta_{I} & 0 & 0 \\ \eta_{U} & 0 & \eta_{I} & 0 \\ \eta_{V} & 0 & 0 & \eta_{I} \end{pmatrix}, \text{ where }$$

$$\eta_I = \frac{1}{2} \eta \sin^2 \gamma + \frac{1}{4} (\eta^+ + \eta^-) (1 + \cos^2 \gamma)$$

$$\eta_Q = \left(\frac{1}{2}\eta - \frac{1}{4}(\eta^+ + \eta^-)\right) \sin^2 \gamma \cos 2\phi$$

$$\eta_{U} = \left(\frac{1}{2}\eta - \frac{1}{4}(\eta^{+} + \eta^{-})\right)\sin^{2}\gamma\sin 2\phi$$

$$\eta_V = \frac{1}{2} \left(\eta^+ - \eta^- \right) \cos \gamma$$

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Transfer of Polarized Light

$$\cos\theta \frac{d\boldsymbol{I}}{d\tau} = (\boldsymbol{1} + \boldsymbol{\eta})(\boldsymbol{I} - \boldsymbol{B}_{\lambda})$$

Longitudinal magnetic field

$$\gamma = 0 \implies \eta_Q = 0, \ \eta_U = 0, \ \eta_I = \frac{1}{2} (\eta^+ + \eta^-), \text{ and } \ \eta_V = \frac{1}{2} (\eta^+ - \eta^-)$$

Radiative transfer equations

$$\mu \frac{dI}{d\tau} = (1 + \eta_I)(I - B_{\lambda}) + \eta_V V \qquad \mu \frac{dQ}{d\tau} = (1 + \eta_I)Q$$

$$\mu \frac{dV}{d\tau} = (1 + \eta_I)V + \eta_V(I - B_{\lambda}) \qquad \mu \frac{dU}{d\tau} = (1 + \eta_I)U$$

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Transverse magnetic field

$$\gamma = 90^{\circ}$$
 and $\phi = 0 \implies \eta_U = \eta_V = 0$ and $U \equiv 0$

$$\eta_I = \frac{1}{2}\eta + \frac{1}{4}(\eta^+ + \eta^-)$$

$$\eta_{\mathcal{Q}} = \frac{1}{2}\eta - \frac{1}{4}(\eta^{+} + \eta^{-})$$

Radiative transfer equations

$$\mu \frac{dI}{d\tau} = (1 + \eta_I)(I - B_{\lambda}) + \eta_Q Q$$

$$\mu \frac{dQ}{d\tau} = (1 + \eta_I)Q + \eta_Q (I - B_{\lambda})$$

$$\mu \frac{dV}{d\tau} = (1 + \eta_I)V$$

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