

# Physics 681: Solar Physics and Instrumentation – Lecture 8

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## Spectroscopy

- ❑ Grating Spectrograph
- ❑ Lyot Filter
- ❑ Fabry-Pérot Interferometer
- ❑ Please check your TurnItIn.com account!

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# Grating Spectrograph

- Collimate light beam on spectrograph slit

$$\frac{D}{f} = \frac{1}{N}$$

- Focal ratio of collimator

$$\frac{f_s}{D_s} = \frac{f}{D}$$

- Grating equation

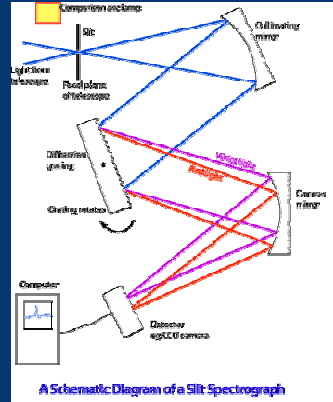
$$m\lambda = a(\sin \alpha + \sin \beta) \quad \text{with } \beta = \beta(\lambda)$$

- Angular dispersion

$$\frac{d\beta}{d\lambda} = \frac{m}{a \cos \beta}$$

- Linear dispersion

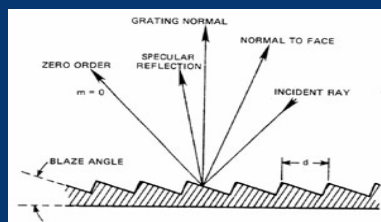
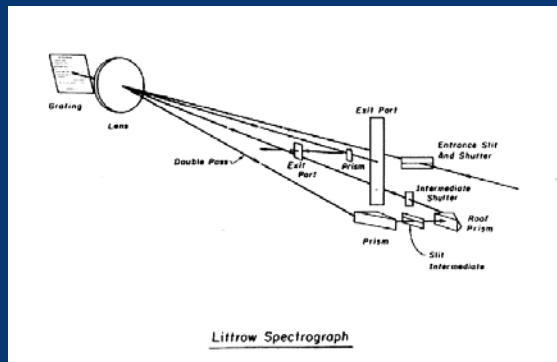
$$\frac{dx}{d\lambda} = f_s \frac{d\beta}{d\lambda} = f_s \frac{m}{a \cos \alpha} \quad \text{with } \beta \approx \alpha$$



<http://outreach.atnf.csiro.au/education/senior/astrophysics/spectrograph.html>

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# Lyot Filter

- Birefringent filter
- Alternating sequence of polarizers and birefringent crystals
- Polarizers are mounted parallel to each other
- Uniaxial crystals cut parallel to their optical axis
- Optical axis of crystals and polarizers make a 45° angle
- Thickness of the  $n$ th crystal plate

$$e_n = 2^{n-1} e \quad \text{with } n = 1, 2, \dots$$

- Phase difference and birefringence

$$\delta = 2\pi e(n_o - n_e) / \lambda \quad \text{with } J = n_o - n_e$$

- Interference of two waves passing a single element

$$\frac{A}{2} \cos(\phi + \delta) + \frac{A}{2} \cos \phi = A \cos(\delta/2) \cos(\phi + \delta/2)$$



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- Intensity

$$I = A^2 \cos^2(\delta/2) \quad \text{with } \delta_{\max} = 2k\pi \quad \text{and } \delta_{\min} = (2k-1)\pi$$

- Transmission maxima  $\lambda = eJ / k$

- Separation  $\simeq eJ / k^2$  for large  $k$

- $N$  elements between  $N + 1$  polarizers

$$I = A^2 \cos^2(\pi eJ / \lambda) \cos^2(2\pi eJ / \lambda) \cos^2(4\pi eJ / \lambda) \dots \cos^2(2^{N-1} \pi eJ / \lambda)$$

- Individual transmission maxima at

$$\lambda = eJ / k, \quad \lambda = 2eJ / k, \quad \lambda = 4eJ / k, \quad \dots \quad \lambda = 2^{N-1} eJ / k$$

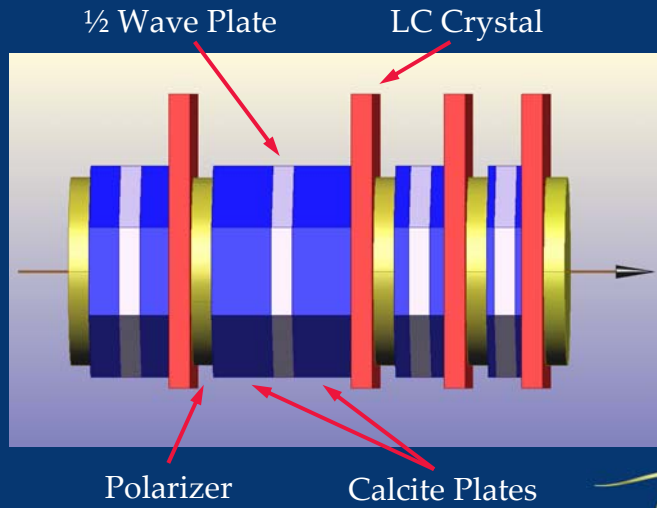
- Central wavelength  $\lambda = 2^{N-1} eJ / k$



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# NIR Lyot-Filter Design



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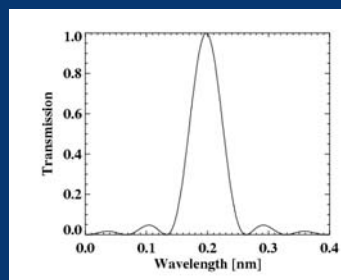
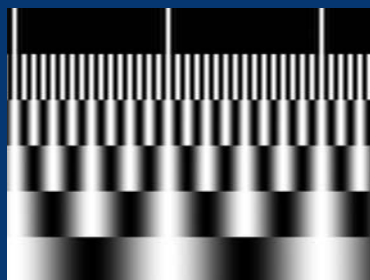


- Band width

$$\Delta\lambda = 2^{N-1} eJ \left( \frac{1}{k-1/2} - \frac{1}{k+1/2} \right) \approx 2^{N-1} eJ / k^2$$

$$\Rightarrow 2^{N-1} e = \lambda^2 / J \Delta\lambda$$

- Birefringence (calcite  $J = -0.172$  and quartz  $J = 0.0092$ )



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# Fabry-Pérot Interferometer

- Phase difference

$$\delta = 2\pi\Delta / \lambda = 4\pi nd \cos \theta / \lambda \quad \text{with } \Delta = 2nd \cos \theta$$

- Outgoing wave

$$Ae^{i\omega t} = Te^{i\omega t} + TRe^{i(\omega t + \delta)} + TR^2e^{i(\omega t + 2\delta)} + \dots$$

$$A = T(1 + Re^{i\delta} + R^2e^{2i\delta} + \dots) = \frac{T}{1 - Re^{i\delta}}$$

- Intensity

$$I = AA^* = \frac{T^2}{1 - 2R \cos \delta + R^2} = \frac{T^2}{(1 - R)^2 + 4R \sin^2(\delta/2)}$$

$$I = I_{\max} \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\delta}{2}} \quad \text{with } I_{\max} = \frac{T^2}{(1-R)^2}$$



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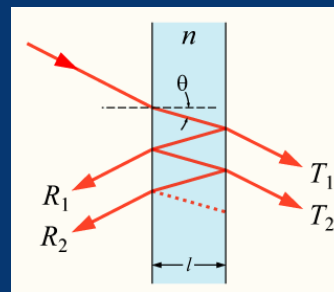
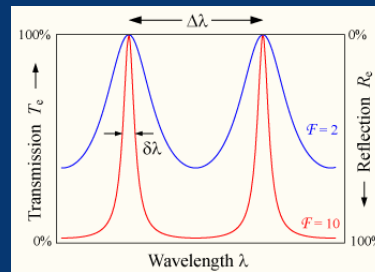
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- Band width

$$\Delta\lambda = FSR / \mathfrak{F}$$

- Finesse

$$\mathfrak{F} = \frac{\pi\sqrt{R}}{1-R}$$



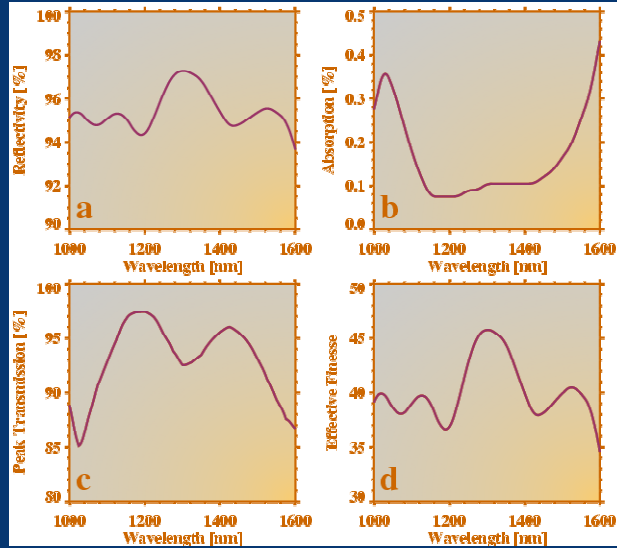
<http://www.physicsdaily.com/physics/Fabry-Perot>



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# Coating Curves

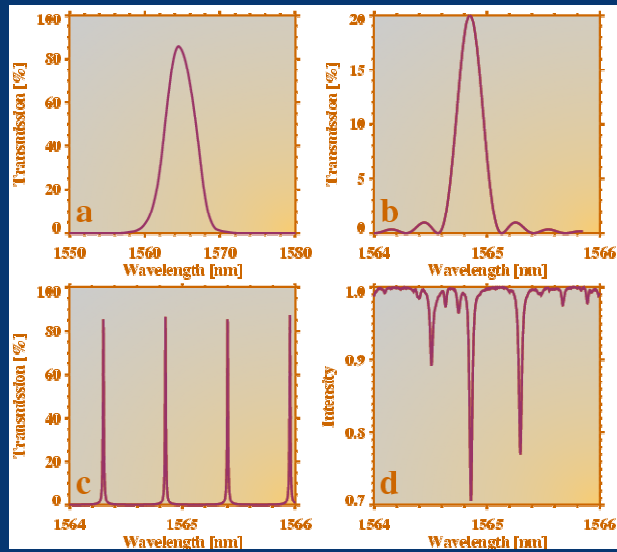


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# Filter Transmission and Spectrum



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