

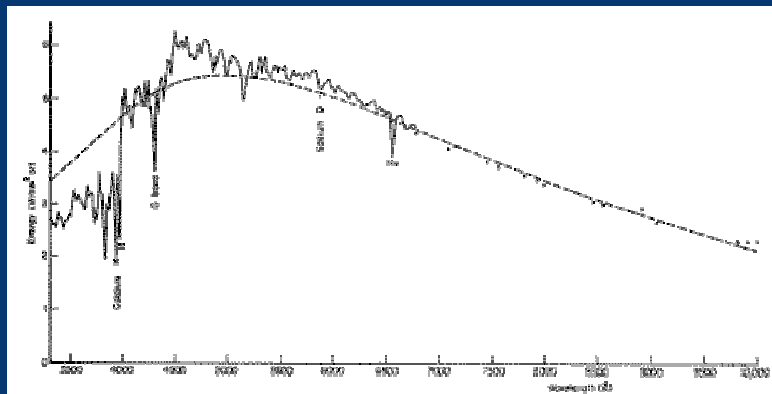
Physics 681: Solar Physics and Instrumentation – Lecture 4

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Solar Spectrum



The spectrum of the Sun: The dashed line is the curve of an ideal blackbody having the Sun's effective temperature.



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Radiation Field

$$\bar{I}_\lambda = \frac{E_\lambda d\lambda}{d\lambda dt dA \cos\theta d\Omega} \quad (\text{average intensity}) \quad \left[\frac{\text{J}}{\text{s m}^3 \text{ sr}} \right]$$

$$\Rightarrow E_\lambda d\lambda = I_\lambda d\lambda dt dA \cos\theta d\Omega = I_\lambda d\lambda dt dA \cos\theta \sin\theta d\phi$$

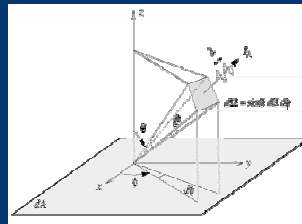
$$\langle I_\lambda \rangle \equiv \frac{1}{4\pi} \int I_\lambda d\Omega = \frac{1}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi \quad (\text{mean intensity})$$

isotropic radiation field

$$\Rightarrow \langle I_\lambda \rangle = I_\lambda$$

isotropic blackbody radiation

$$\Rightarrow \langle I_\lambda \rangle = B_\lambda$$



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Energy Transport

- Radiation and convection
- Energy flux (luminosity per unit area): $F = F_R + F_C = L / 4\pi r^2$
- Mean free path of photons is small in the solar interior \Rightarrow local thermal equilibrium (LTE)
- Radiative transfer equation

$$\cos\theta \frac{dI_\nu}{dr} = -\kappa_\nu \rho (I_\nu - B_\nu) \quad \text{with} \quad B_\nu(T) = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1}$$

- Total energy flux

$$F = \int_0^\infty F_\nu d\nu \quad \text{with} \quad F_\nu = \int \cos\theta I_\nu d\nu = -\frac{4\pi}{3\kappa_\nu \rho} \frac{dB_\nu}{dr} = -\frac{4\pi}{3\kappa_\nu \rho} \frac{dB_\nu}{dT} \frac{dT}{dr}$$

- Temperature gradient

$$\frac{\partial T}{\partial m} = \frac{3\kappa L}{256\pi^2 \sigma r^4 T^3} \quad \text{with} \quad \frac{1}{\kappa} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$



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Temperature Definitions

excitation temperature
(Boltzman equation) $\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b-E_a/kT)}$

ionization temperature
(Saha equation) $\frac{N_{i+1}}{N_i} = \frac{2kTZ_{i+1}}{P_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2}$ and $P_e = n_e kT$

kinetic temperature
(Boltzman distribution) $n_v dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv$

color temperature
(Planck function) $B_\lambda(T) = \frac{2hc^2}{\lambda^5} \left(\frac{1}{e^{hc/\lambda kT} - 1} \right)$



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Opacity

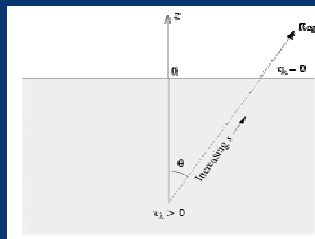
Any process that removes photons from a beam of light will be called **absorption**.

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds \quad (\kappa_\lambda \text{ [m}^2 \text{ kg}^{-1}\text{] absorption coefficient or opacity)}$$

$$d\tau_\lambda = -\kappa_\lambda \rho ds \quad (\text{optical depth})$$

$$\Delta\tau_\lambda = \tau_{\lambda,f} - \tau_{\lambda,i} = -\int_0^s \kappa_\lambda \rho ds$$

$$0 - \tau_{\lambda,0} = -\int_0^s \kappa_\lambda \rho ds \Rightarrow \tau_\lambda = \int_0^s \kappa_\lambda \rho ds$$



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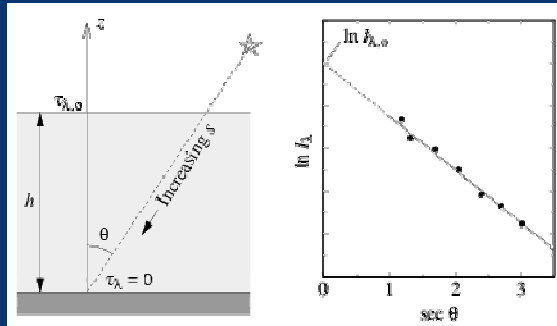
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The optical depth may be thought of as the number of mean free paths from the original position to the surface, as measured along the ray's path.

$$I_\lambda = I_{\lambda,0} e^{-\tau_\lambda} \quad (\text{pure absorption})$$

$$\tau_\lambda \gg 1 \quad (\text{gas optically thick})$$

$$\tau_\lambda \ll 1 \quad (\text{gas optically thin})$$



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Radiative Transfer Equation

$$dI_\lambda = j_\lambda \rho ds \quad (j_\lambda [\text{m s}^{-3} \text{ sr}^{-1}] \text{ emission coefficient})$$

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho ds$$

$$\Rightarrow -\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda}$$

$$S_\lambda \equiv \frac{j_\lambda}{\kappa_\lambda} \quad ([\text{J s}^{-1} \text{ m}^{-3} \text{ sr}^{-1}] \text{ source function})$$

$$\Rightarrow -\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda \quad (\text{transfer equation})$$

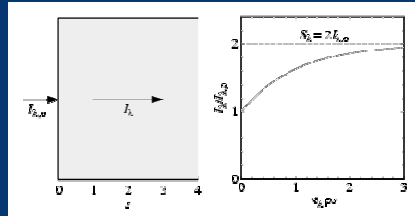
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The intensity of the light tends to become equal to the local value of the source function.

For the case of thermodynamic equilibrium, the source function is equal to the Planck function $S_\lambda = B_\lambda$.



$$\frac{dI_\lambda}{ds} = 0 \Rightarrow I_\lambda = S_\lambda$$

$$\frac{dI_\lambda}{ds} < 0 \Rightarrow \text{intensity decreases with distance}$$

$$\frac{dI_\lambda}{ds} > 0 \Rightarrow I_\lambda \text{ approaches } S_\lambda$$



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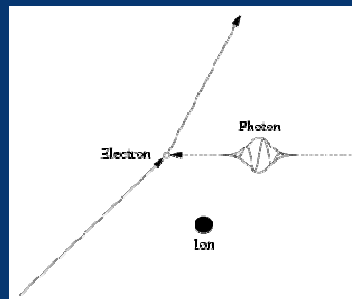
Photon Interactions

- Bound-bound transitions
- Bound-free absorption
- Free-free absorption (bremsstrahlung)
- Electron scattering

$$\sigma_{bf} = 1.31 \times 10^{-19} \frac{1}{n^5} \left(\frac{\lambda}{500 \text{ nm}} \right)^3 \text{ m}^3$$

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.65 \times 10^{-29} \text{ m}^2$$

(Thompson scattering)



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Continuum Opacity

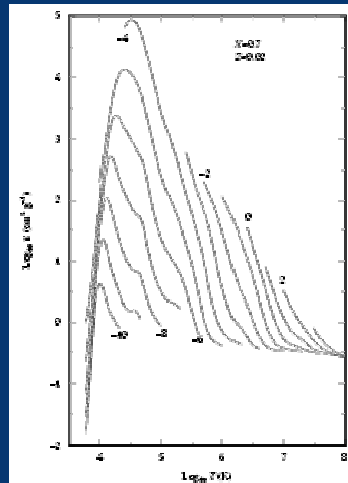
Photoionization of H⁻ ions

$$\lambda \leq \frac{hc}{\chi} = \frac{1240 \text{ eV nm}}{0.754 \text{ eV}} = 1640 \text{ nm}$$

$$\kappa_{\lambda} = \kappa_{\lambda,bb} + \kappa_{\lambda,bf} + \kappa_{\lambda,ff} + \kappa_{\lambda,es}$$

(total opacity)

The total opacity depends not only on the wavelength of the light being absorbed but also on the composition, density, and temperature of the stellar material. ⇒ Rosseland mean opacity



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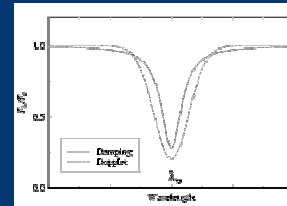
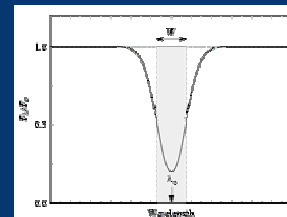
Structure of Spectral Lines

$$W = \int \frac{F_c - F_{\lambda}}{F_c} d\lambda \quad (\text{equivalent width})$$

natural broadening $\Delta\lambda \approx \frac{\lambda^2}{2\pi c} \left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f} \right)$

Doppler broadening $\Delta\lambda = \frac{2\lambda}{c} \sqrt{\frac{2kT}{m}}$

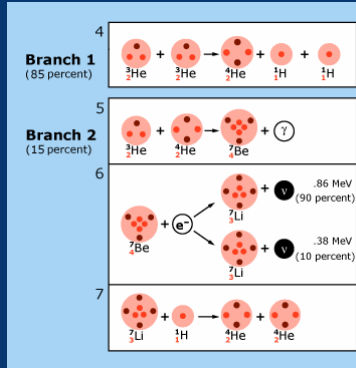
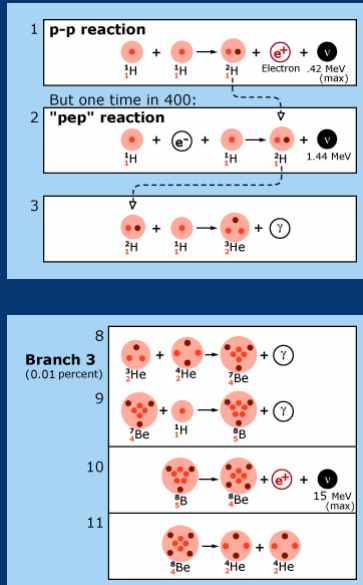
pressure broadening $\Delta\lambda = \frac{\lambda^2}{c} \frac{1}{\pi \Delta t_0} \approx \frac{\lambda^2}{c} \frac{n\sigma}{\pi} \sqrt{\frac{2kT}{m}}$



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Nuclear Reactions



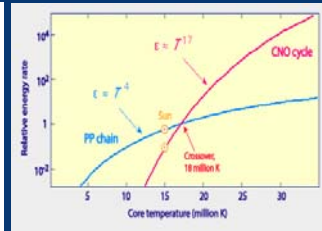
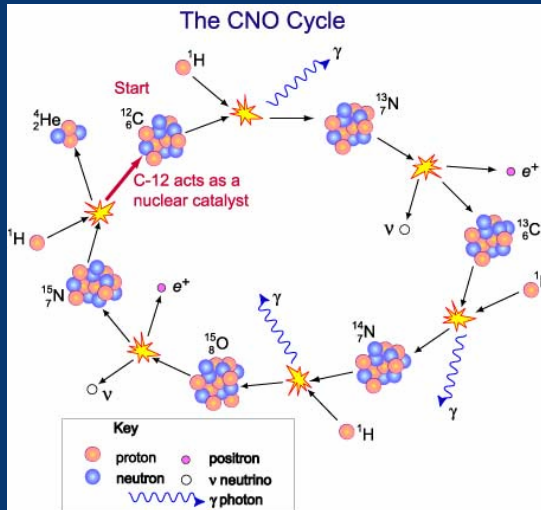
http://nobelprize.org/physics/articles/fusion/sun_pp_chain.html



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CNO Cycle



http://outreach.atnf.csiro.au/education/senior/astrophysics/stellarevolution_mainsequence.html

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