

# Physics 681: Solar Physics and Instrumentation – Lecture 3

*Carsten Denker*

*NJIT* Physics Department  
Center for Solar–Terrestrial Research



## Internal Structure

- Construction of a Model
  - The Evolutionary Sequence
  - The Standard Model
- Age and Pre-Main-Sequence Evolution
- Model Ingredients
  - Conservation Laws
  - Energy Transport
  - Element Diffusion and the Interior
  - The Entropy
  - Nuclear Energy Sources
  - The Opacity
  - Boundary Conditions and Method of Solution

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## Construction of a Model

- ❑ How can we study the interior of the Sun?
  - Solar neutrinos
  - Helioseismology
  - Theory! (use of general physical concepts and principles, conservation of energy or mass, chemical composition, magnetic or centrifugal forces, ...)
- ❑ Goals for a model of stellar structure:
  - Explain the Hertzsprung-Russell diagram of star clusters
  - Match the observed characteristics of the Sun (mass, radius, luminosity, age, ...)
  - Describe the evolutionary sequence from a proto-star to the present day Sun



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## The Evolutionary Sequence

- ❑ Energy: hydrogen  $\Rightarrow$  helium conversion
- ❑ How much helium has already been synthesized in the interior?
- ❑ Helium/hydrogen ratio increases from model to model
- ❑ Thermal equilibrium: total emitted radiation  $\Leftrightarrow$  total amount generated helium
- ❑ Mean molecular weight increases
- ❑ Density and temperature of the core has to increase to support the weight of the star
- ❑ Nuclear reaction rates increase
- ❑ Luminosity increases
- ❑ Start with a chemically homogeneous star of one solar mass
- ❑ Initial fractional Helium content by weight  $Y_0 = 0.25$  (shortly after "Big Bang")
- ❑ Evolve model until we get today's luminosity ( $3.844 \times 10^{26}$  W)  
 $\Rightarrow$  age ( $4.6 \times 10^9$  yr, from radioactive decay of isotopes in meteorites)



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- How about the radius ( $6.96 \times 10^8$  m)?
- Convective energy transport in the outer layers of the Sun  $\Rightarrow$  "Mixing-Length Theory"

$$\alpha = l / H_p \quad \text{with} \quad H_p = -1 / (d \ln P / dr)$$

- $\alpha=1$ : A parcel of convecting gas travels a distance of the order of the local pressure scale height before dissolving.

$$\ln L = \ln L_{\odot} + a(Y_0 - Y_{0\odot}) + b(\alpha_0 - \alpha_{0\odot})$$

$$\ln r = \ln r_{\odot} + c(Y_0 - Y_{0\odot}) + d(\alpha_0 - \alpha_{0\odot})$$

$$a \equiv \frac{\partial \ln L}{\partial Y_0} = 8.6 \quad b \equiv \frac{\partial \ln L}{\partial \alpha} = 0.04$$

$$c \equiv \frac{\partial \ln r}{\partial Y_0} = 2.1 \quad d \equiv \frac{\partial \ln r}{\partial \alpha} = -0.13$$

- Original Helium content:  $Y_{0\odot} = 0.276$

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## The Standard Model

- Spherical symmetry
- All parameters depend only on  $r$
- Internal rotation is sufficiently slow
- Internal magnetic fields are sufficiently small
- Fractional abundance by weight:  
 $X + Y + Z = 1 \quad (Z = 0.02)$
- CNO energy generation rate  $\Rightarrow$  initial  $^{12}\text{C}$  and  $^{14}\text{N}$  abundances in the core
- Heavy elements  $\Rightarrow$  the Sun is a second-generation star (very old stars in globular clusters)
- Convection and mixing  $\Rightarrow$  Schwarzschild criterion has been violated
- Radiation and convection in unstable layers and radiation in stably stratified layers (heat conduction)

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## Age and Pre-Main-Sequence Evolution

- ❑ Radioactive dating of meteorites  $\Rightarrow (4.55 \pm 10^9 \text{ yr})$
- ❑ How is the age of meteorites related to the ignition of hydrogen burning, i.e., the start of the Sun's main-sequence life?
- ❑ Star formation
- ❑ Interstellar cloud ( $10^4 m_{\odot}$ ) is triggered into gravitational collapse
- ❑ Shock waves in galactic spiral arms or supernova explosion ( $^{26}\text{Mg}$  isotope)
- ❑ Jeans criterion: 
$$\frac{Gm_c}{r} > \frac{\mathcal{R}T}{\mu}$$
- ❑ Collapse takes about  $3 \times 10^7 \text{ yr}$
- ❑ Magnetic field and rotation (removal of angular momentum, magnetic braking)
- ❑ Fragmentation (star formation)
- ❑ Accretion of material in  $10^6 \text{ yr}$  to form a proto-star (hydrostatic equilibrium,  $T_{\text{eff}} = 3000 \text{ K}$ )



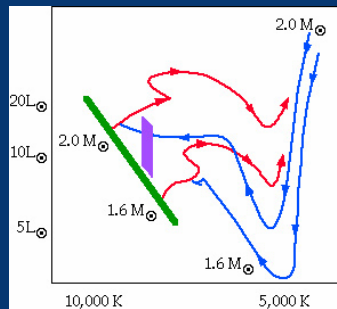
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- ❑ "Hayashi Line" (fully convective stars) in Hertzsprung-Russell diagram
- ❑ Luminosity  $\Rightarrow$  gravitational energy  $\Rightarrow$  virial theorem
- ❑ Slow contraction phase "Kelvin-Helmholtz Time"

$$t_{\text{KH}} = \frac{Gm^2}{rL} \approx 3 \times 10^7 \text{ yr}$$

- ❑ Zero-Age Main-Sequence (ZAMS, chemically homogenous stars)
- ❑ This is about the time when the meteorites condensed
- ❑ Age of the Sun:  $t_{\odot} = (4.57 \pm 0.05) \times 10^9 \text{ yr}$
- ❑ Both luminosity and radius of the Sun have increased during the main sequence evolution to reach today's values.



<http://www.tim-thompson.com/hr.html>



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## Model Ingredients

- Laws and equations which govern solar structure and evolution including boundary conditions.

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}$$

$$\rho = \rho(P, T) \quad dS = dS(P, T)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

$$\varepsilon = \varepsilon(\rho, T) \quad \kappa = \kappa(\rho, T)$$

$$\frac{\partial L}{\partial m} = \varepsilon - T \frac{\partial S}{\partial t}$$

$$\frac{\partial T}{\partial m} = \begin{cases} -\frac{3\kappa L}{256\pi^2\sigma r^4 T^3} & \text{(in a stable layer)} \\ (\partial T / \partial m)_c & \text{(in an unstable layer)} \end{cases}$$



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## Conservation Laws

- Conservation of mass ( $m = m_\odot$ ):  $\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}$

- Hydrostatic equilibrium:  $\frac{\partial P}{\partial r} = -\rho g$  and  $g = -\frac{Gm}{r^2}$

- Conservation of momentum:  $\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$

- Hydrodynamic balance (proto-star collapse or oscillations):

$$\frac{\partial^2 r}{\partial t^2} = -4\pi r^2 \frac{\partial P}{\partial m} - \frac{Gm}{r^2}$$

- Energy balance:  $\frac{\partial L}{\partial m} = \varepsilon - T \frac{\partial S}{\partial t}$



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